Effect of Refraction on the Setting Sun as Seen from Space in Theory and Observation

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The theory of refraction predicts that the setting sun or moon as seen from space should be highly flattened. The Mercury Project Manned Space Flights MA-6 and MA-7 have provided photographs of the phenomenon. To compare theory with observation for an observer situated outside the atmosphere an essentially new approach to the refraction problem is employed which is closely related to the theory of refraction height as given by Chauvenet. Theoretical solar profiles for four true zenith distances of the center of the setting sun were constructed, which may be compared to the photographs taken by Glenn and Carpenter.

THE problem of the refraction of light by the earth's atmosphere as seen from a space capsule differs essentially from the problem as seen from the surface of the earth. At the earth's surface it is possible to calculate the astronomical refraction within 1 second of arc by Comstock's formula down to elevation angles of 15° above the horizon. Comstock's formula depends only on the elevation angle and the index of refraction at the observer; it would be the same if the earth were flat and the atmosphere a hundred feet thick. Near the horizon, it is true that terms involving the scale height and the curvature of the earth must be introduced. The problem of the vertical displacement of the ray by refraction is hardly considered, except in certain eclipse calculations.

In the case of the capsule, on the other hand, owing to the great distance (of the order of 1000 km) from the observer to the relevant region of the atmosphere, the variation of the refractive index with path is an essential part of the computation. We cannot approach the problem without a good knowledge of the scale height and of the curvature of the earth. The vertical displacement of the ray is relatively enormous.

The observation of the rising and setting of the sun in Mercury Project manned orbital flights has emphasized the need for a more complete theory. It is found that the solar image should appear strongly flattened, almost sausage-shaped. Astronauts Glenn and Carpenter obtained photographs of the setting sun that illustrate the rather striking effect.

A general procedure is presented for the computation of refraction for extreme altitudes in order to construct a theoretical solar profile for comparison with the actual photographs. Application is made to Carpenter's orbital conditions of 24 May 1962. The quantities to be determined are the apparent zenith distance and the true zenith distance, as seen from the capsule, denoted by Z_{app} and Z_{true}, respectively. To find these quantities a ray through the atmosphere to the capsule is idealized. The phenomenon takes place effectively only for rays whose perigees are lower than 20 km above the surface of the earth. Thus rays were considered at two-kilometer intervals up to 20 km altitude (h). Figure 1 illustrates the geometry of the situation.

In Fig. 1, the ray from the sun is traced backward from the capsule, C. The first section, from the spacecraft to the atmosphere, X, is straight. If the ray continued in this direction toward the sun, there would be a point B of nearest approach to the center of the earth O. That distance is denoted by p, and the angle at the center of the earth from the capsule to B as Θ. If B and p are known, the apparent height of any point on the sun as seen from the spacecraft could be calculated.

To make the calculation, the curving optical ray is followed forward until it is refracted so as to be parallel to the surface of the earth. This point is called the perigee of the ray, and is denoted by G. The line OG makes an angle $\Theta+r$ with OC; and here r is the refraction angle for the sun when an observer (fictitious) at G sees it 90° from the zenith.

If the straight portion of the ray is prolonged, it will intersect OG at some point such as D. Then the height of D above G is called the refractive height s. For any given height, say G, the refraction angle r at the horizon and the refractive height s, which depends on the true height and r, can be calculated. Then the right triangle OBD can be solved for the distance p from the center of the earth to the straight-line prolongation of C, the space portion of the ray. The length p is denoted, by
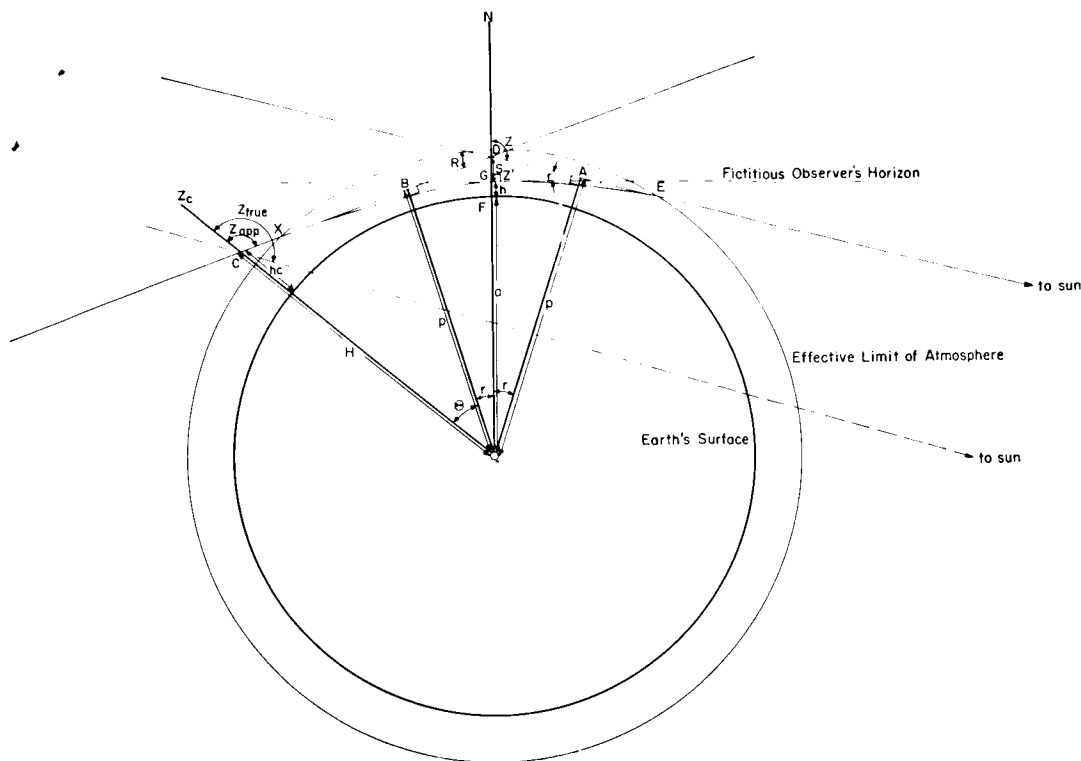
FIG. 1. Geometry of ray from setting sun as seen from the capsule.

analogy with the similar dynamical problem, as the impact parameter.

Given p, and the capsule height, the apparent angles at the spacecraft can be calculated as a function of Θ. The refraction angle $R = 2r$ is added to form the true zenith distances.

The computation of the refraction $r = z - z'$, where z is the true zenith distance and z' the apparent zenith distance, respectively, for the fictitious observer stationed at perigee, was based on the rather detailed theory of Garfinkel (1944, method I), the only one suitable for use at zenith distances 90° or greater. The pertinent formulas are

$$r = T^{\frac{1}{2}} \sum_{i=0}^{5} B_i W^{i+1},$$

$$\cot\theta = \gamma T^{-\frac{1}{2}} \cot z,$$

$$W = P T^{-2},$$

where $\theta = 90°$; T is the absolute temperature divided by 273°.0 at height h; P is the pressure at height h divided by the ground pressure of 1.013×10^6 dyn/cm²; the B's are coefficients involving the index of refraction μ and the polytropic index n and for $z = 90°$ are: $B_0 = 2012''.2$, $B_1 = 168''.2$, $B_2 = 21''.8$, $B_3 = 3''.2$, $B_4 = 0''.5$; and γ is a constant dependent on n. The temperature, pressure, and density δ of the atmosphere at altitude h were taken from the Rocket Panel (1952) data. More recent

data on T, P, and δ are available from CIRA (1961), but the results on this computation are not significantly different.

For greater accuracy than required here, corrections to the approximations used in Garfinkel's method may be made by the formula

$$E = \epsilon_1 + \epsilon_2 + \epsilon_3 + \epsilon_4 = F_1 p T^{-\frac{1}{2}} + (F_2 h R / 10^4) + \epsilon_3$$

$$+ T^{\frac{1}{2}} \csc\theta \sum_{i=0}^{\infty} J_i [w \tan^2(\tfrac{1}{2}\theta)]^{i+1} \delta n,$$

where F_1, F_2, J_0, J_1, J_2 are tabulated as functions of θ, and ϵ_3 is tabulated as a function of $|90° - z|$ and h. Note that there is a factor of 10^4 omitted in Garfinkel's paper and that ϵ_2 should be computed as given here.

The parameter s, here called the refractive height, is a refraction correction commonly applied in calculations of times of contact in eclipses. The derivation of s is to be found in Chauvenet (1960, p. 515). Equation (564) there gives its relation to the index of refraction as $1 + s/a = \mu \sin z' / \sin z$, where a is the mean radius of the earth (6 371 020 m); μ is the index of refraction at h; z' is the apparent zenith distance (90°) for a fictitious observer at G; and z is the true zenith distance ($z' + r$) at the same point.

Once μ, r, and s have been obtained, then p is obtained from the equation $p = (a + h + s) \cos r$. Then Θ is determined from the relation $\cos\Theta = p/H$, where

TABLE I. Summary of computed results.

h (m)	T	P	δ^a	r	μ	s (m)	p	Θ	Z_{app}	Z_{true}
0 000	1.0000	1.0000	1.0000	36.765	1.0002944	2240.1	6 372 896	15°949	105°949	107°175
2 000	1.0330	0.7932	0.8532	27.081	1.0002512	1797.9	6 374 620	15.894	105.894	106.797
4 000	0.9985	0.6214	0.6903	22.073	1.0002032	1425.8	6 376 314	15.841	105.841	106.577
6 000	0.9524	0.4812	0.5611	18.188	1.0001652	1141.7	6 378 072	15.785	105.785	106.391
8 000	0.8974	0.3676	0.4551	15.092	1.0001340	915.5	6 379 873	15.728	105.728	106.231
10 000	0.8454	0.2757	0.3623	12.299	1.0001067	720.6	6 381 699	15.669	105.669	106.079
12 000	0.8040	0.2038	0.2819	9.742	1.0000830	554.3	6 383 548	15.610	105.610	105.935
14 000	0.7751	0.1488	0.2128	7.468	1.0000626	413.5	6 385 419	15.550	105.550	105.799
16 000	0.7619	0.1075	0.1567	5.508	1.0000461	302.0	6 387 313	15.489	105.489	105.673
18 000	0.7656	0.0775	0.1124	3.922	1.0000331	214.7	6 389 231	15.426	105.426	105.557
20 000	0.7795	0.0562	0.0802	2.757	1.0000236	152.3	6 391 170	15.363	105.363	105.455

[a] δ is the density at h divided by the density at the surface $(1.172 \times 10^{-3}$ g/cm$^3)$ and is tabulated for use in the computation of μ.

$H = a + h_c$; $h_c = 257\,000$ m as determined by the orbit computed from the final definitive elements of Carpenter's orbit. Finally, Z_{app} and Z_{true} are related to Θ and R such that $Z_{app} = 90° + \Theta$ and Z_{true} $= 90° + (\Theta + R)$. Table I summarizes the computed results.

The flattening of the image of the setting sun may be illustrated by the use of a plot of Z_{app} vs Z_{true}. An
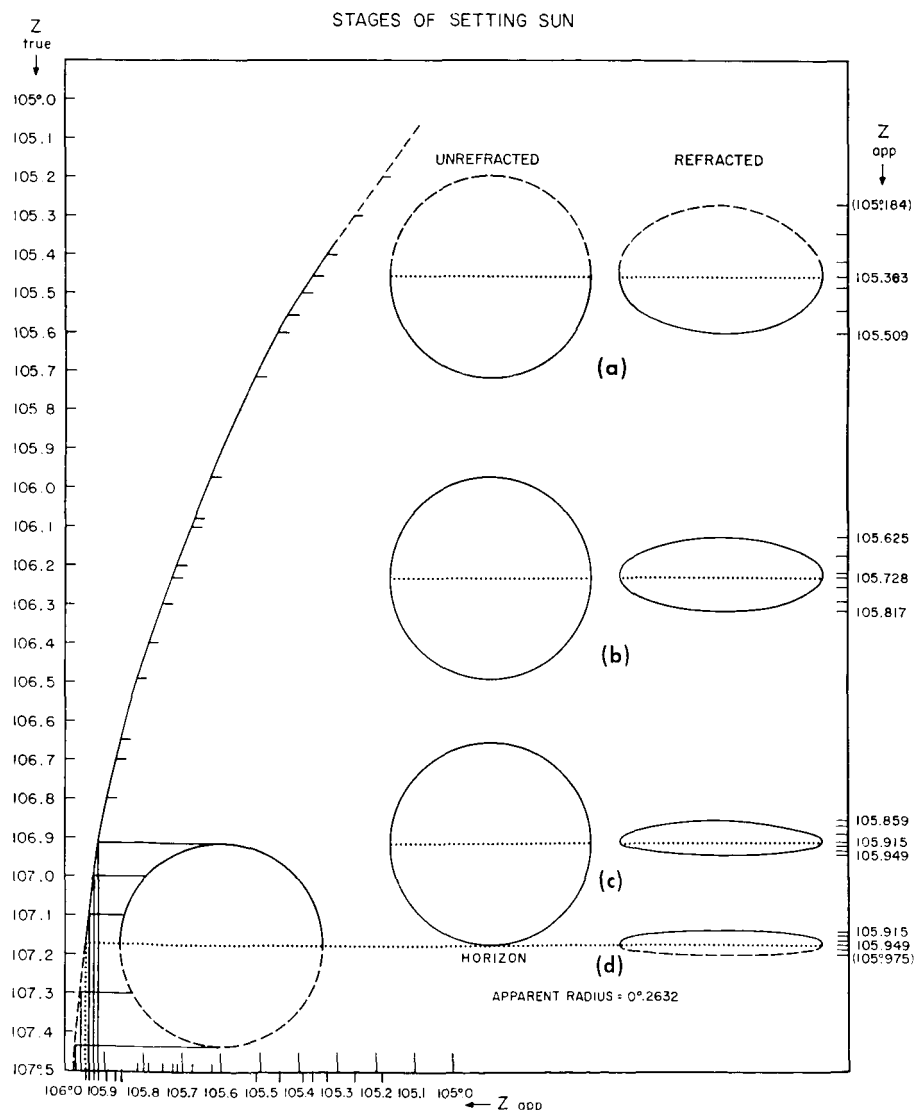

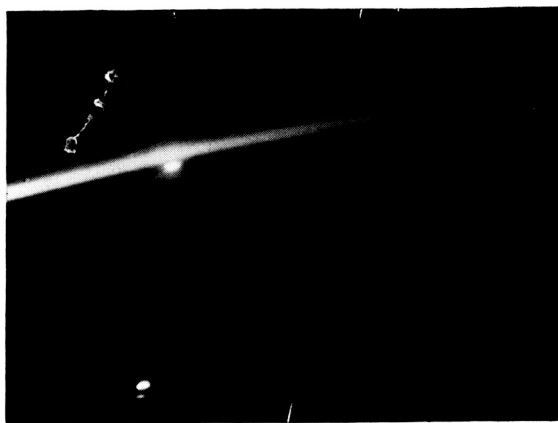FIG. 2. Stages of the setting sun for four zenith distances.

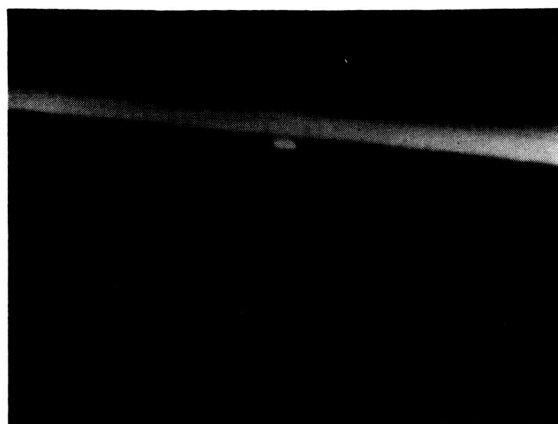FIG. 3. Photograph of setting sun taken by Glenn
on the MA-6 orbital flight.

FIG. 4. Photograph of setting sun taken by Carpenter
on the MA-7 orbital flight.

image representing the sun to scale may be placed at any Z_{true}, and points around the limb, extended to the curve, may be located on the Z_{app} axis, giving the apparent zenith distance of those points. Since the horizontal axis is not affected by refraction, parallels of altitude (almucantars) may be laid off on the un-refracted image of the sun, and similarly laid off on the apparent image of the sun. The latter may be rectified for easy comparison. The theoretical profiles of four phases of a setting sun are illustrated in Fig. 2. This is a plot of Z_{true} vs Z_{app} with images of the sun given for four true zenith distances of the sun's center: $(2a)$ $Z_{true} = 105°.455$; $(2b)$ $Z_{true} = 106°.231$; $(2c)$ $Z_{true} = 106°.915$ (sun's lower limb on the horizon); and $(2d)$ $Z_{true} = 107°.175$ (sun's center on horizon). The ratio of the vertical to horizontal diameters are approximately 0.63, 0.36, 0.17, and 0.11, respectively. Considering the capsule angular velocity of $4°/\text{min}$, it is seen that the entire effect treated here would take place in the relatively short interval of about 20 sec of time for the astronaut.

The uncertainty in times of photography precludes an exact comparison of theory and observations. However, $(2c)$ perhaps most nearly simulates the photographs in Figs. 3 and 4, which show the effects of the capsule's motion by increasing the vertical diameter of the image somewhat but still demonstrate the effect. Figure 3 was photographed on the MA-6 orbital flight of 20 February 1962. The sun was not seen then as a narrow, flat object, but instead was observed to spread out about ten degrees on either side and to merge with the twilight band. In Fig. 3, the true setting sun and horizon appear in the center of the picture and the reflections occur below. The phenomenon perhaps can be seen more clearly in the lower reflection.

Figure 4 was photographed on the MA-7 orbital flight on 24 May 1962. At that time the sun was observed to be definitely flattened during sunrise and sunset, very much like the appearance of the sun in the photographs. The flattening effect of refraction on a setting celestial object as seen above the atmosphere, a condition simulated by the capsule in orbit, has been demonstrated by direct observation. However, it is hoped that future missions will obtain photographs with precise times of observation; and perhaps measures of the apparent vertical and horizontal diameters with a sextant are feasible observations.

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